Quantization of Special Relativity

Abstract

This paper discusses the need to quantize special relativity in order to obtain a complete theory of gravity. Previously, a quantized version of the famous equation, $E = mc^2$, was introduced as presented in [1]. Einstein's relativity assumes a continuous space-time structure and is a classical, not quantum, theory. A quantization of the Lorentz transform, however, would lead to quantization of space-time as well as mass as would be necessary to obtain a complete model of quantum gravity using a quantized metric.

Gregory Charles Matthews
Toronto, ON
M4E 2T5

The theory of Special Relativity as first presented by Einstein [2] has to date been verified in depth for high energy state particles and bodies. However, failure to harmonize relativity with quantum theory has led to crisis in theoretical physics. The experimental proof of Bell's Inequality [3], [4] has shown phenomena that apparently exhibit space-like geodesics in contradiction with Einstein.

The theory of Special Relativity has its roots in classical physics evolved to high energy or speed frames of reference. One of the shortcomings of Einsteinian relativity is that it does not lend itself easily as a quantum theory. This may be summarized by saying that while we have obtained a relativistic quantum theory, a quantized relativity theory has remained elusive.

In [1], a quantized version of the famous relativistic equation is presented, using *vector ∗ product* and vector division notation using a double divisor introduced in [1] and [6], as:

$$-i\hbar\nabla_\mu * (\phi_\mu // v_\mu) = \frac{E_\mu}{c^2} * \phi_\mu = m_\mu * \phi_\mu \tag{1}$$

A quantization of the Lorentz transform may be similarly developed by taking:

$$v_\mu = \frac{hk_\mu}{\overline{m_\mu}}\text{ , } v = \frac{hk}{m},\ E = h\nu,\ \frac{v}{c} = \frac{hk}{mc}\text{ , and } E_\mu = cp_\mu \tag{2}$$

The Lorentz transform may then be expressed as:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - h^2k^2/m^2c^2}} \tag{3}$$

where $k^2 = \| k_\mu * k_\nu^\dagger \|$, noting that the expression of **k** as a complex term is unusual but necessary in a rigorous treatment of relativity for possible tachyonic, or space-like, behaviors.

The primary result of the Lorentzian transform was that as v —> c, a singularity arises in the expression for time dilation, making space-like geodesics forbidden. A singularity of this type may, however, be addressed as in [4], by applying the Dirac delta function of $\delta(x-a)$ and a step function to eliminate the singularity. We write:

$$\tau = \int \delta(c-v)\ h(v)\gamma t dv = t, \text{ for } v\ = c \qquad (4)$$

where the step function, h(v) is expressed as:

$$h(v) = \left[\begin{array}{l} 1,\ v < c, \text{ or } v > c \\ 0^{1/2},\ v = c \end{array}\right]$$

yielding an expression for proper time with a quantized Lorentz transform. Similarly, with respect to spatial contraction, we write:

$$x' = \int \delta(c-v)\ h'(v)\gamma^{-1} x dv = x, \text{ for } v\ = c, \text{ where} \qquad (5)$$

$$h'(v) = \left[\begin{array}{l} 1,\ v < c, \text{ or } v > c \\ 0^{-1/2},\ v = c \end{array}\right]$$

Equations (4) and (5) imply that, for the usually forbidden case of an observer in a light-like frame of reference, the spatial and temporal dimensions would not be contracted or dilated.

One may also note that upon quantization of the Lorentz transform, effectively a quantization of spatial and temporal coordinates is achieved. We may then substitute the mass term using the quantized Lorentzian into the Alembertian as follows:

$$(\ \Box - (\gamma m_0)^2)\ \phi = 0 \qquad (6)$$

Of interest, with the elimination of the singularity arising at v = c in the Lorentz transform, treatment of the transform in this manner would eliminate the barrier to space-like geodesics needed to allow for Bell's inequality. Furthermore, with quantization of mass and particle speed, asymptotic behavior implied by the Lorentz transform at v = c, may be overcome. This approach would require modeling the metric with complex spatial and temporal coordinates as proposed in [1], as may be represented on a Minkowski diagram using complex coordinates for x and t.

One of the standard objections to faster than light travel is that applied to Dirac mass bearing particles, the resultant mass would become imaginary. This may be resolved by re-expressing the basic parameters assumed by Einsteinian Special Relativity in complex form such as, for $m\ \varepsilon\ \mathscr{I}$, $v\ \varepsilon\ \mathscr{I}$, the $p\ \varepsilon\ \boldsymbol{R}$, and we may obtain $iE = imc^2$.

Another argument often made is that if a body were to achieve a speed greater than c, the electromagnetic bonds binding the atomic structure of the body would be broken. Advances made in quantum mechanics after publication of the theory of Special Relativity produced results that are in apparent disagreement with this theory, and with the prohibition of non-local behavior at the foundation of Special Relativity. Atomic bonding, for example, uses an electronic cloud model where the electrons appear to be simultaneously in many different loci. As a result, the argument made that the atomic bonds would be broken for v > c would fail. Similarly, the famous double slit experiment also appears to contradict the assumption of non-locality and the prohibition of space-like behavior. Non-locality is further contradicted by the verification of Bell's Inequality for behaviors of entangled particles [8], [9]. These well-known phenomena described by quantum mechanics should compel quantization of Special Relativity.

Suggesting that $\{t, x,\} \varepsilon C$, we may also include a term for ict usually refuted in text books on Relativity by stating a form, $icit = - ct$, which may be further re-expressed and applied to the generally disallowed term, ict as:

$$\frac{(1 + i)}{\sqrt{2}}c \, \frac{(1 - i)}{\sqrt{2}}t = ct \tag{7}$$

and similarly, its converse adjoint:

$$\frac{(i - 1)}{\sqrt{2}}c \, \frac{(i + 1)}{\sqrt{2}}t = -ct \tag{8}$$

A simple interpretation of the expression of these terms would be that they represent a phase shift as a rotation on an Argand diagram representation of a Minkowski plot. Complex coordinate systems to model space and time would be required for any model of a quantized metric. One may also note that any quantization of the metric of General Relativity will have, by definition, the general form:

$$g_{\mu\nu} = \phi_\mu \phi_\nu^\dagger \tag{9}$$

which implies the existence, mathematically, of an anti-metric of the form:

$$g_{\mu\nu}(-) = i\phi_\mu \, i\phi_\nu^\dagger = - g_{\mu\nu} \tag{10}$$

An anti-metric of the form in (7), that is + - - -, as opposed to that of equation (10), - + + +, has generally been dismissed as necessarily non-existent because it results in space-like geodesics not allowed by classical Special Relativity.

Of interest, with the elimination of the singularity arising at $v = c$ in the Lorentz transform, treatment of the transform in this manner would eliminate the barrier to space-like geodesics needed to allow for Bell's inequality. Furthermore, with quantization of mass, and of particle speed, asymptotic behavior implied by the Lorentz transform at $v = c$, may be overcome. A model for this behavior could be obtained by constructing the metric with complex spatial and temporal coordinates as proposed in [1] and [5].

With respect to the value of c itself, a proper and rigorous quantization of Maxwell's equations would be expected to yield eigenvalues for c, as suggested in [1]. Those eigenvalues should show as a tiny spread of peaks in a plot of c vs. E where the spread of peaks would lie within statistical error of the normalized distribution established for the existing experimental determinations of the speed of light. Use of a highly accurate and sensitive detection apparatus would be required to detect such peaks of the values of c. The fine splitting of spectral lines for sensitive measures of eigenvalues of c may support an explanation of the value of the fine structure constant.

With respect to the classical high energy physics of Special Relativity, consider, as a *gedanken* experiment, a particle subject to two transverse fields sufficient to accelerate the particle along both transverse axes of the fields. We do the *gedanken* experiment by first accelerating the particle along a field in the x-component direction to the point where the particle reaches a speed of $v_x = 0.9$ c, and then, after the particle exits the x-directional field, we subject the particle to a second transverse field along the y-component direction of so that $v_y = 0.9c$ relative to the frame of reference of the particle at $v_x = .9c$ after acceleration through the first field, so that one can ask if the speed of the particle is given by $v = \sqrt{v_x^2 + v_y^2}$. Substitution of v into the Lorentz transform would result in the finding that $L \in \mathscr{I}$ for a subset of particle velocities in curvilinear motion.[1]

With the results of Equation (4), relativistic frames of reference with $v \geq c$ may be understood as non-relativistic within their own frame of reference, (i.e., observed value for v is $v \ll c$) but from another frame of reference of another observer as $v \geq c$. This, together with [1], could allow for the prediction of tachyons and other non-physical particles that arise in string theories. Such particles may only be allowed if metrics can be expressed with complex coordinates to include propagation along imaginary coordinates. Otherwise, a mathematical form for elimination of non-physical particles may be obtained by making a modification to the expression for h(v) of Equation (4) to the step-function so that $h(v) = 0$, where $v \geq c$, as shown in Equation (10). The test for whether or not space-like geodesics are encountered would rest on any discovery of Cerenkov radiation in empty space, and on detection of time and particle mass violations of Einstein.

If experiments were to arise that demanded a model of the metric in complex coordinates, such as evidence of space-like geodesics, we could interpret apparent “tachyonic behaviours” as rotational phasing on an Argand diagram representation of a Minkowski plot. It may be further added that for physics models relying on $D > 4$, such as Superstring theory and M-theory, the additional dimensional representations may be understood as the result of the imaginary terms that would be used in a complex coordinate representation of space and time.

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[1] We note that the standard approach would be to say that the Lorentz transform would forbid a particle speed in excess of c, as formulated here. The point contested is that an observer, O_2, in the same frame of reference of the particle at $v_x = 0.9c$ after exiting the x-directional field, would be expected to observe the speed of the particle as zero prior to the admission of the particle to a transverse field along the y-direction. *Ergo*, from the observer O_2’s point of view, a particle speed of $v_y = 0.9c$ after admittance of the particle to the transverse field would be allowed. From the original frame of reference of an observer, O_1, of the particle before admittance to either field, what would the Lorentz transform yield as the final speed of the particle?

References:

1. G. C. Matthews, *A Proposed Treatment of Quantum Vector Field Theory*, unpublished, April 18, 2010.
2. Einstein, A, Relativity, the Special and the General Theory, Crown, New York, 1961.
3. D. Bouwmeister, J. Pan, et. al., *Nature* **390,** 575 (1997).
4. D. Boschi, S. Branca, et. al., *Phys. Rev. Lett.,* 80, 1121, (1998).
5. G. C. Matthews, *Use of Complex Variables in Tensor Analysis,* unpublished, May 2, 2011.
6. G. C. Matthews, *Application of Scalar-Vector Transform to Quantum Theory*, unpublished, May 30, 2011.
7. G. C. Matthews, *A Note on Einstein's Twin Paradox Applied to the Behavior of Entangled Particles,* unpublished, February 1, 2011.
8. D. Boumeister et. al., Nature 390, pp. 575-9, 1997.
9. W. Tittel et. al., *Experimental Confirmation of Quantum Correlations over more than 10 kilometers*, Physical Review, A57, (3229), 1998.